Formal Proofs for the Security of Signcryption

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Signcryption

- Proposed by Zheng at Crypto '97
- Provides both message confidentiality and authenticity (non-repudiation & unforgeability) *in an efficient way*
- Has received a lot of attention
  - a number of papers about signcryption have been published
  - Submitted to standard committee P1363

Security of signcryption

- However, *formal proofs* for the security of signcryption have not been provided
- Formal proofs
  - "formal proofs" = "reductions from attacking the signcryption scheme to solving computationally difficult problems"
  - To provide formal proofs of security, first of all we need to establish a *sound security model for signcryption*

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What we have achieved

- A sound security model for signcryption:
  - ***Flexible public key model***
    - encompassing CCA security (security against adaptive chosen ciphertext attack)
  - Attackers in our model are allowed to be very powerful!

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What we have achieved (cont.)

- Proofs for the confidentiality and unforgeability of signcryption
  - Confidentiality --- Providing a reduction
    - from breaking CCA security of signcryption with respect to the flexible public key model
    - to breaking the ***GAP Diffie-Hellman assumption*** in the ROM (Random Oracle Model)
  - Unforgeability --- Providing a reduction
    - from breaking unforgeability of signcryption against CMA (Chosen Message Attack)
    - to Discrete Logarithm problem in the ROM

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Difference between our model and previous models

- Motivation
  - An attacker can produce her own public key and replace Alice and/or Bob's public keys to break the confidentiality or authenticity
  - Therefore, the security model of encryption + authentication in asymmetric setting should be different from that in the symmetric setting

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Difference between our model and previous models (cont.)

- Security model for encryption + authentication (E+A) in the symmetric setting
  - Formalized by Bellare & Namprepre at Asiacrypt 2000 [BN]
  - Only Encryption-then-MAC (EtM) composition is CCA-secure

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Difference between our model and previous models (cont.)

- Observation:
  - Results on confidentiality in the symmetric setting are NOT applicable to E+A in the asymmetric setting.
  - Specifically, Encrypt-then-Sign (EtS, the corresponding *simple* asymmetric version) *is completely insecure against CCA!*

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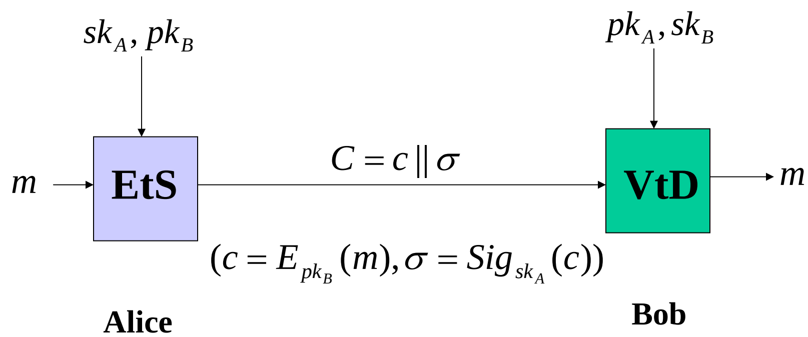
CCA attack on the simple EtS

- Simple EtS

Alice's private/public key : (sk_A, pk_A)

Bob's private/public key: (sk_B, pk_B)



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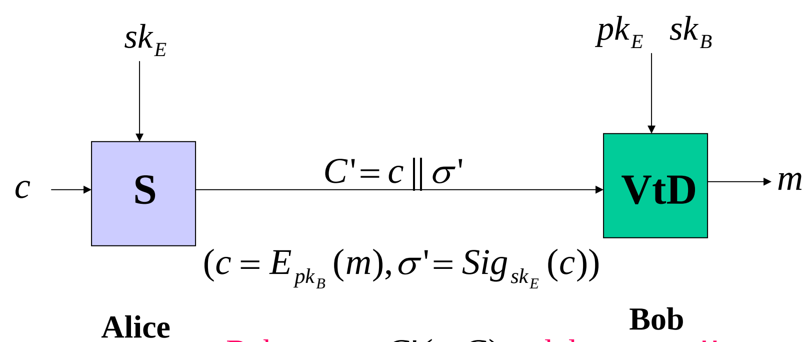
CCA attack on the simple EtS

- Attack

Eve's private/public key: (sk_E, pk_E)

Bob's private/public key: (sk_B, pk_B)



Bob accepts $C'(\neq C)$ and decrypts c !!

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Signcryption: an EaS variant

- Signcryption may be viewed as a variant of the simple EaS (Encrypt-and-Sign) composition.
  - It employs 'EaS' concept to gain efficiency
- However, signcryption is NOT merely a simple EaS scheme!
  - It fixes, intuitively, the problem that the simple EaS composition is not *generically secure* (since the signature part can reveal some information about plaintext as observed in [BN])

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Flexible Public Key model

- Flexible Unsigncryption Oracle (FUO) model
  - Public key input for the unsigncryption oracle is *flexibly* given

Normal Unsigncryption Oracle: $USC^{G(.),H(.)}_{y_A,x_B}(.)$

Flexible Unsigncryption Oracle: $USC^{G(.),H(.)}_{x_B}(.)$

No specific sender's public key is given

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FUO-IND-CCA2

- Confidentiality notion for signcryption with respect to adaptive chosen ciphertext attack (CCA2) under semantic security
- A CCA attacker has access to
  - – the Flexible Unsigncryption Oracle, and
  - – (fixed) Signcryption Oracle
    - (to be extended to flexible signcryption oracle (FSO) model in our forthcoming paper)

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Another tool

- GAP Diffie-Hellman problem
  - – Proposed by Okamoto & Pointcheval at PKC '01
  - – Attacker searches the Diffie-Hellman key $g^{xy} \text{ mod } p$ of $g^{x} \text{ mod } p$ and $g^{y} \text{ mod } p$ with the help of a decisional Diffie-Hellman Oracle,

$$DDH(g, g^x, g^y, W) = \begin{cases} 1 & \text{if } W = g^{xy} \bmod p \\ 0 & \text{otherwise} \end{cases}$$

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Another tool (cont.)

- The GAP-DH problem is hard as long as there is no reduction from the DDH problem to the CDH (Computational DH) problem (-> The GAP-DH assumption)

- With the help of the DDH oracle, the flexible unsigncryption/signcryption oracles can be successfully simulated

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Another tool (cont.)

- Actually, the GAP DH assumption is a *necessary condition* for some CCA-secure schemes to be proven (in our forthcoming paper)

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“bind” information

- “*bind*” info contains the sender Alice’s public key y_A and the receiver Bob’s public key y_B
  - It was pointed out by Zheng that this bind info should be included in the input to hash function H(.) to thwart “double spending attack”
  - This observation was crucial, as the “bind” information turned out to be necessary in proving the confidentiality of signcryption.

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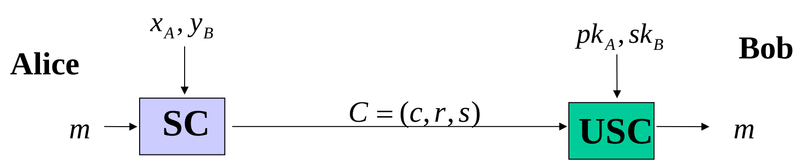
Signcryption scheme that we used in our formalization

Alice’s private/public key: $(x_A, y_A(= g^{x_A} \bmod p))$

Bob’s private/public key: $(x_B, y_B(= g^{x_B} \bmod p))$ $\quad bind = y_A \| y_B$



Signcryption

$c = ESYM_\tau(m)$,

$r = H(m \| bind \| \kappa)$,

$s = x/(r + x_A) \bmod q$

where $\tau = G(y_B{}^x \bmod p)$

$\kappa = y_B{}^x \bmod p$

UnSigncryption

$m = DSYM_\tau(c)$

if $H(m \| bind \| \kappa) = r$

where $\tau = G((y_A g^r)^{s x_B} \bmod p)$

$\kappa = (y_A g^r)^{s x_B} \bmod p$

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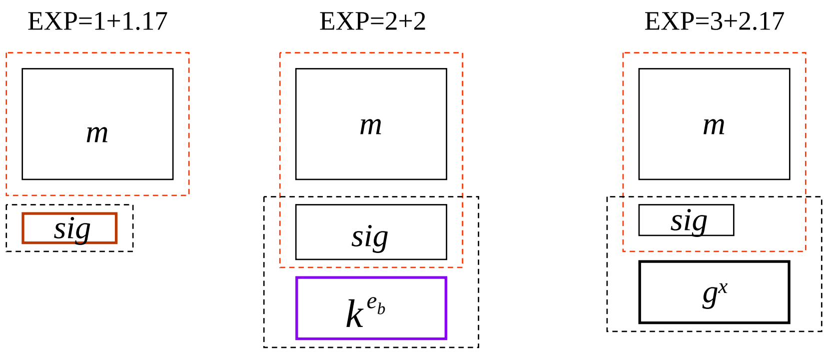
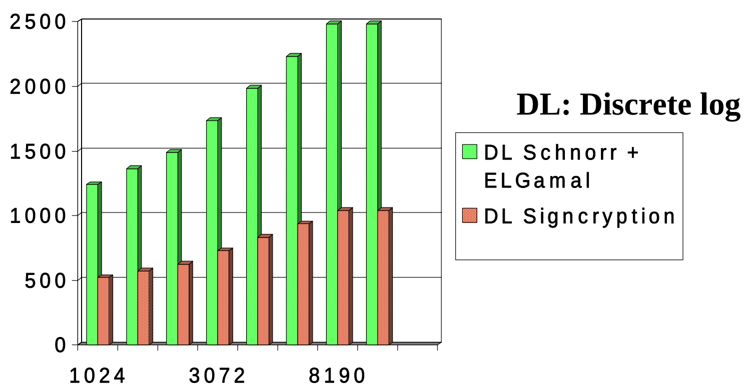
Signcryption v.s. Signature-then-Encryption



(a) Signcryption based on DL
(b) Signature-then-Encryption based on RSA
(c) Signature-then-Encryption based on DL

Time --- DL Signcryption v.s. DL Signature-then-Encryption

Time -- # of multiplications

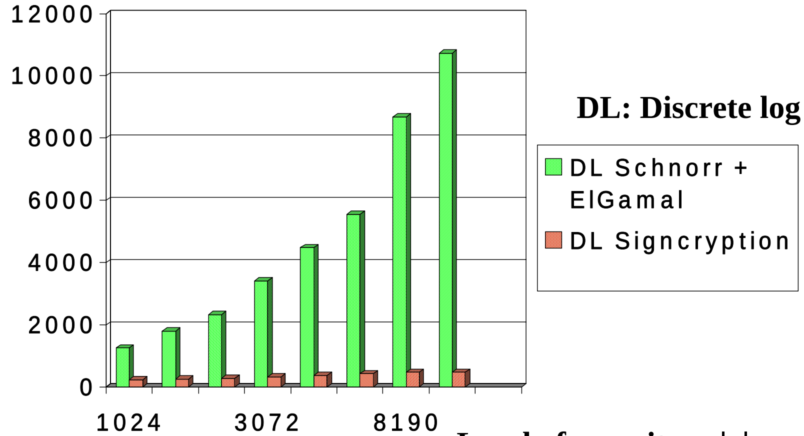


Level of security -- |*p*|

Bandwidth --- DL Signcryption v.s. DL Signature-then-Encryption

Comm. overhead -- # of bits

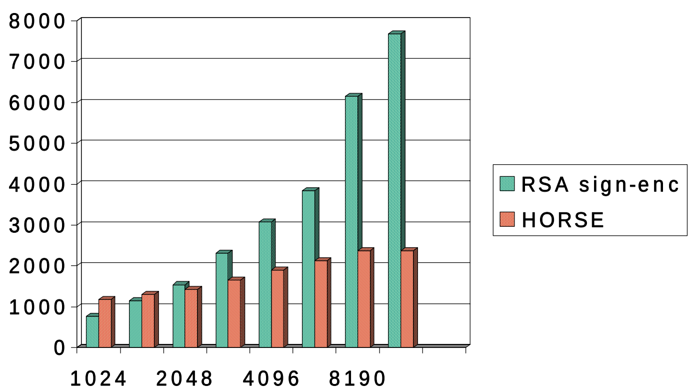
DL: Discrete log



Level of security -- |*p*|

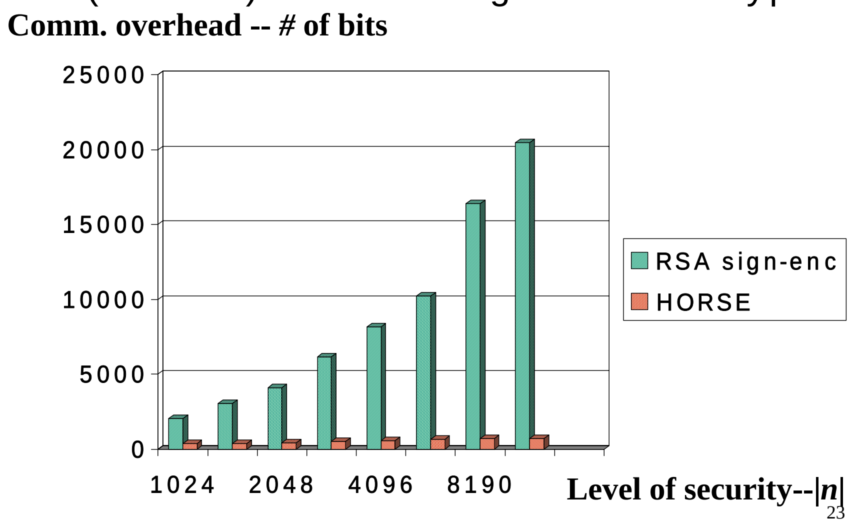
Time -- RSA signcryption (HORSE) v.s. RSA sign-then-encrypt

Time -- # of multiplications



Level of security--|*n*|

Bandwidth -- RSA signcryption (HORSE) v.s. RSA sign-then-encrypt



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Confidentiality --- Sketch of proof

- An attacker (or an attack algorithm) for the GAP DH problem A_{gdh} runs adaptive chosen ciphertext attacker A_c to find the DH key $g^{xy} \bmod p$, given $g^x \bmod p$ and $g^y \bmod p$
- It is assumed that the A_c has access to the flexible unsigncryption oracle as well as the signcryption oracle
- The random oracles G and H, the signcryption/flexible unsigncryption oracle are successfully simulated with the help of the DDH oracle

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Confidentiality --- Sketch of proof (cont.)

- When the events **Bad** and **GDHBrk** do not happen, we can construct a chosen plaintext attacker A_p which uses A_c as subroutine
  - **Bad**: The event which causes the distribution of A_c's view to differ in experiment in the simulation from the distribution of A_c's view in the real attack
  - **GDHBrk**: The event that A_c asks the DH key g^{xy} mod p to the random oracle G or A_c asks a query *h* to the random oracle H where the k-rightmost bits of *h* is the DH key

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Confidentiality --- Sketch of proof (cont.)

- As a result, we obtain the following upper bound:

$$\mathbf{Adv}_{\mathsf{SC}}^{\mathrm{fuo-ind-cca2}}(k, t, q_g, q_h, q_{sc}, q_{usc}) \le 4\mathbf{Adv}_{\mathsf{GDH}}^{\mathrm{invert}}(k, t_1, q_{ddh}) + \mathbf{Adv}_{\mathsf{SC}^{\mathrm{SYM}}}^{\mathrm{ind-cpa}}(l, t_2, 0) + \frac{q_{sc}(q_g + q_h + 1) + q_{usc}}{2^{l_q(k)-1}}$$

All the variables are defined in our PKC02 paper

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Confidentiality --- Sketch of proof (cont.)

- Maim Theorem 1:
  Signcryption is secure
  - against adaptive chosen ciphertext attacks
  - in the random oracle model
  - assuming the GAP Diffie-Hellman Problem is hard

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Security notion for unforgeability of signcryption

- Follows the security notion for unforgeability of signcryption formulated by Steinfeld and Zheng (ISW '00)
- Allows the forger to have access to Bob's private key as well as the corresponding public key
  - Since signcryption offers non-repudiation for the sender Alice, it is essential that even the receiver Bob cannot impersonate Alice and forge valid signcrypted text from Alice to himself

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Unforgeability --- Sketch of proof

- Convert a forger F which mounts chosen message attack on the signcryption scheme into an passive attacker A_i for the identification scheme derived from the signcryption scheme
- An attacker A_{dlp} for discrete logarithm problem uses A_i to solve the discrete logarithm associated with Alice's public key. (i.e., we use the ID-reduction technique by Ohta & Okamoto (Crypto '98))

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Unforgeability --- Sketch of proof (cont.)

- As a result, we obtain the following upper bound:

$$\mathbf{Adv}_{\mathsf{SC}}^{\mathrm{cma}}(k, t, q_g, q_h, q_{sc}) \leq 2q_h\left(\mathbf{Adv}_{\mathsf{DLP}}^{\mathrm{search}}(k, t^*)\right)^{\frac{1}{2}} + \frac{1}{2^{l_q(k)}}$$

All the variables are defined in our PKC02 paper

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Unforgeability --- Sketch of proof (cont.)

- Maim Theorem 2:
  Signcryption is existentially unforgeable
    - against adaptive chosen message attacks
    - in the random oracle model
    - assuming the Discrete Logarithm Problem is hard

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Future work

- Providing the confidentiality proof using *FSO + FUO model*
- Providing the security proofs for various signcryption schemes proposed so far, including
  - Steinfeld-Zheng scheme (ISW ’00) based on integer factorization problem
  - Zheng scheme (PKC ’01) based on higher residuosity problem
  - Others …

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Thank you very much!

감사합니다.

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