Spin Singlet Stability for 1/2 Spin Heisenberg Spin Rings

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Overview

- The Heisenberg Model is one of the most important models used in the study of magnetic systems.
- The Heisenberg Model considers solely interactions between nearest-neighbor particles in a system. These interactions are due to their wavefunctions overlapping, which leads to the creation of a magnetic momentum.
- By obtaining the eigenvectors of the matrix used to represent this system, one can find the energy levels of specific
- In our work, we provide numerical evidence for the presence of local minima in energy levels of 1D self-connected quantum spin systems, which is relevant for the development of superconductors and artificial magnets.

Background

Our work is based on the Lieb-Mattis theorem and its representation of the ground state of antiferromagnet in a spin singlet, which shows that the minimum energy increases monotonically with increasing spin. However, this does not inherently suggest that ferromagnets ought to follow the same pattern. Sutherland has shown using the Bethe ansatz that the Heiseinberg ferromagnet does not exhibit this monotone energy increase. Using perturbation theory as presented in a standard quantum spin system textbook, one can generate numerical evidence for this behavior.

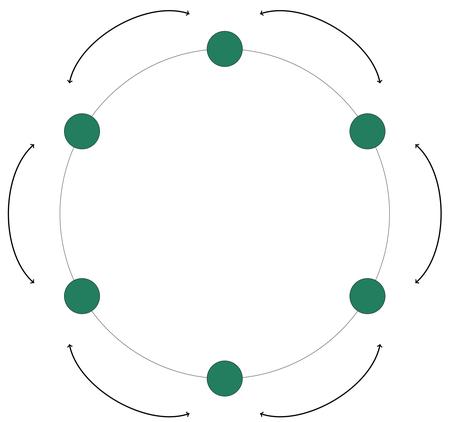
Methods

In a spin-j Heisenberg ferromagnet ring of an even-length L, the Hamiltonian is a sum over the sites L of $h^{(j)}_{k,k+1}$, where

$$h^{(j)}_{k,k+1} = (-S^{(1)}_k S^{(1)}_{k+1} - S^{(2)}_k S^{(2)}_{k+1} - S^{(3)}_k S^{(3)}_{k+1} + j^2\,\mathbf{1})$$

with $\mathbf{1}$ being the Hilbert space identity matrix. For $j = 1/2$ the matrices are the usual variations of the Pauli spin-1/2 matrices. We consider both the Standard Basis and the Hulthen Bracket Basis to calculate the eigenvalues and eigenvectors of these systems using C++ Armadillo and Matlab, the computation themselves consisting mostly of running Lancsoz-algorithms on large sparse matrices. We have been able to simulate systems of up to $L = 22$ sites. One of the biggest bottlenecks in this process is the memory requirement for this computation, Hamiltonians of size 2^L by 2^L having to be generated and then decomposed for each system of size L.

Graphical Representation of a L = 6 System

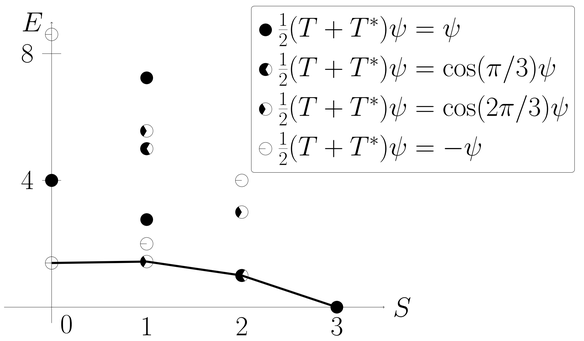


Sample C++ Armadillo Code

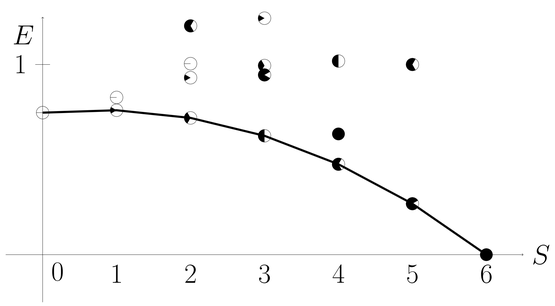
Code used to calculate the spectrum vectors for a Heisenberg Spin Ring using the Standard Basis:

```
mat A = null(Cas);
vec sigma = eig_sym(A.t() * H * A);
vec ss = sort(real(sigma));
int m = 1;
for (int k = 1; k <= floor(L/2); k++) {
K = K + L + 2 - (2 * k);
A = null(Cas - K * eye(powL, powL));
sigma = eig_sym(A.t() * H * A);}
```

Visualized Data



Full spectrum of the spin $j = 1/2$ spin ring for $L = 6$. The line represents $E(S)$. The angles of the arcs represent the translation eigenvalue (pairs).



Full spectrum of the spin $j = 1/2$ spin ring for $L = 12$ (below the cutoff of $E = 1.25$). The line represents $E(S)$. The angles of the arcs represent the translation eigenvalue (pairs).

Conclusion

We have successfully shown by using a simple spin wave operator, spin raising by 1, that the antiferromagnet does not follow the same minimum energy behavior as that of a ferromagnet under the Lieb-Mattis theorem. This demonstrates that simple, standard textbook methods have the capability of solving complex problems such as this one. We currently do not have complete results due to the minuscule effect we observe, but in the future it is hoped systems larger than $L = 22$ could be investigated by implementing more efficient computational algorithms. We are currently working on parallelizing our algorithms to improve computation speed on high-performance clusters, while also investigating quantum Monte-Carlo and machine learning methods to increase computational efficiency.

References

[1] Elliot Lieb and Daniel Mattis.
Ordering energy levels of interacting spin systems.
Journal of Mathemathical Physics, 3:749–751, Jul 1962.

[2] Bill Sutherland.
Low-lying eigenstates of the one-dimensional heisenberg ferromagnet for any magnetization and momentum.
Phys. Rev. Lett., 74:816–819, Jan 1995.

Contact

- Code Repository: `https://github.com/davidcheson/heis_spin_rings`
- Email: `dch376@msstate.edu`

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